

Indian Statistical Institute, Bangalore

B. Math. First Year, First Semester

Probability Theory: Mid-Term Examination

Duration: 3 hours

Maximum marks: 100

Date : 12-09-2012

1. Five persons are chosen at random from 5 couples. What is the probability that there is at least one couple (husband and wife) in them? What is the probability that there are two couples in the chosen 5? [15]
2. An urn contains 4 black balls and 6 red balls. You go on choosing one ball at a time, at random, without replacement, until you get a red ball. Let R be the number of trials (choices) required to get a red ball. Find the probability distribution of R . Write down the probability mass function of R . [15]
3. Let A_1, A_2, A_3 be three mutually independent events in a probability space (Ω, \mathcal{F}, P) with $P(A_1) = \frac{1}{2}, P(A_2) = \frac{1}{3}$, and $P(A_3) = \frac{1}{4}$. Let X_i be the random variable:

$$X_i(\omega) = \begin{cases} 1 & \text{if } \omega \in A_i, \\ 0 & \text{otherwise,} \end{cases}$$

for $1 \leq i \leq 3$. Find the probability distribution, expectation and variance of $S = X_1 + X_2 + X_3$. [20]

4. Let S be the set of all two digit numbers which are multiples of 6. Choose a two digit number XY at random from S . (For example, if the chosen number is 48, then $X = 4$ and $Y = 8$). Find the joint distribution of X, Y . Are X, Y independent? Find variances of X, Y . Find the correlation coefficient between X and Y . [20]
5. We have a population of 1000 people out of which 400 are women and 600 are men. It is known that 50 percent of women and 60 percent of men are literate. A person is chosen at random from the whole population and is found to be literate. What is the probability that the chosen person is a woman? [10]
6. Go on throwing a fair die until you get 'SIX'. Let Z be the number of throws required. Find the probability distribution of Z . Compute the moment generating function of Z . Using the moment generating function compute the expectations of Z and Z^2 . [20]
7. In an admission test there is a multiple choice question paper with 25 questions, where four choices are given for each question with exactly one of them being correct. If a student answers a question correctly he/she gets 4 marks and if the student answers wrongly he/she gets -1 marks for that question. If the student does not answer a question he/she gets 0 marks for that question. Suppose that a student tosses fair coin to decide to answer or not answer a question and if he decides to answer a question he randomly chooses one of the four choices and he/she does this for every question. What is the expected total score of the student? [10]